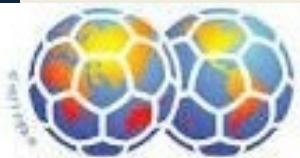


## Overtime Rules

Comparing the fairness of different overtime rules in Association Football

By: Elliot Chanen, Lenny Bronner, and Daniel Ramos



**FIFA**

*For the Good of the Game*

# The Beautiful Game



## World Cup Champions



Loss



## What We Did:

- Compared:
  - regular overtime
  - penalty shoot-outs
  - Golden Goal
- Also looked at Silver Goal, but not enough data
- Wrote recurrence relations for PSO's and Golden Goal
  - Not possible for regular overtime
- Used Position in the FIFA World Ranking

## The Rules - Regular OT

- Regular Overtime:
  - Always two 15 minutes extra time periods



## The Rules - Golden Goal

- Golden Goal:
  - Two times 15 minutes extra time periods
  - Over once the first team has scored

<http://youtu.be/ThpYsN-4p7w>



## The Rules - Silver Goal

- Silver Goal:
  - Two times 15 minutes
  - If teams scores within the first 15 minute half, the game is over after those 15 minutes run out



## The Rules - PKs

- Penalty:
  - Teams alternate taking five penalties
  - Then sudden death



- Gathered data from international games all over the world:
  - Olympics
  - FIFA World Cup
  - Gulf Cup
  - UEFA Euro
  - CONMEBOL Copa America
  - CONCACAF Gold Cup
  - African Cup of Nations
  - AFC Asia Cup
  - FIFA Confederations Cup

## Data

## Olympics

Cup Year	Round	Date	Winner	Loser	Score	OT Rule	Winning Rank	Losing Rank	AET	ASDET	PSO	SILVER GOAL	TOTAL
2012	Quarter-Final	September 4, 2012	Mexico (21)	Senegal (82)	4-2	AET	21	62	WIN				
Classic	Quarter-Final	September 4, 2012	South Korea (74)	Great Britain (3)	1-1, 5-4	PSO	74	3			LOSS		
2008	Quarter-Final	September 16, 2008	Brazil (6)	Cameroon (14)	2-0	AET	6	14	WIN				
Classic		September 16, 2008	Argentina (7)	Netherlands (4)	2-1	AET	7	4	LOSS				
2004	Quarter-Final	September 21, 2004	Italy (9)	Mali (43)	0-1	AET	9	43	WIN				
2000	Quarter-Final	August 23, 2000	United States (21)	Japan (48)	2-2, 5-4	PSO	21	48			WIN		
Classic	Quarter-Final	August 23, 2000	Cameroon (33)	Brazil (1)	1-2	AET	33	1	LOSS				
	Final	August 30, 2000	Cameroon (33)	Spain (5)	2-2, 5-3	PSO	33	5			LOSS		
1996	Quarter-Final	July 27, 1996	Portugal (11)	France (3)	2-1	ASDET	11	3		LOSS			
Golden Goal	Semi-Final	July 31, 1996	Nigeria (47)	Brazil (1)	4-3	ASDET	47	1		LOSS			

SELECT BETWEEN TWO AND FOUR TEAMS

Germany

Argentina

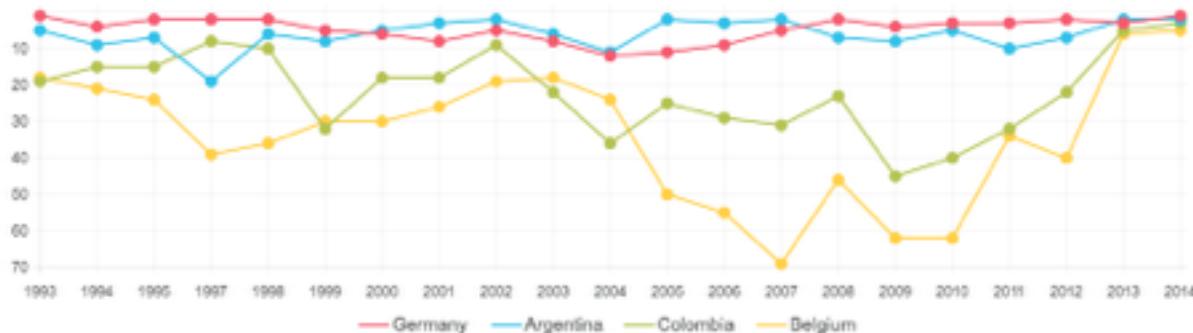
Colombia

Belgium

SELECT A TIME RANGE

Since FIFA Ranking was created

Compare Teams

FIFA  
PUSKÁS  
AWARD 2014VOTE FOR  
THE BEST GOAL »

- Released by the world football organisation: FIFA
- Ranks all 209 teams every month
- Existed since 1993 - start of our data collection
- Much criticized ranking - but best there is!

Ranking points =  $100 \times (\text{Result points} \times \text{Match status} \times \text{Opposition strength} \times \text{Regional strength})$

## FIFA World Ranking

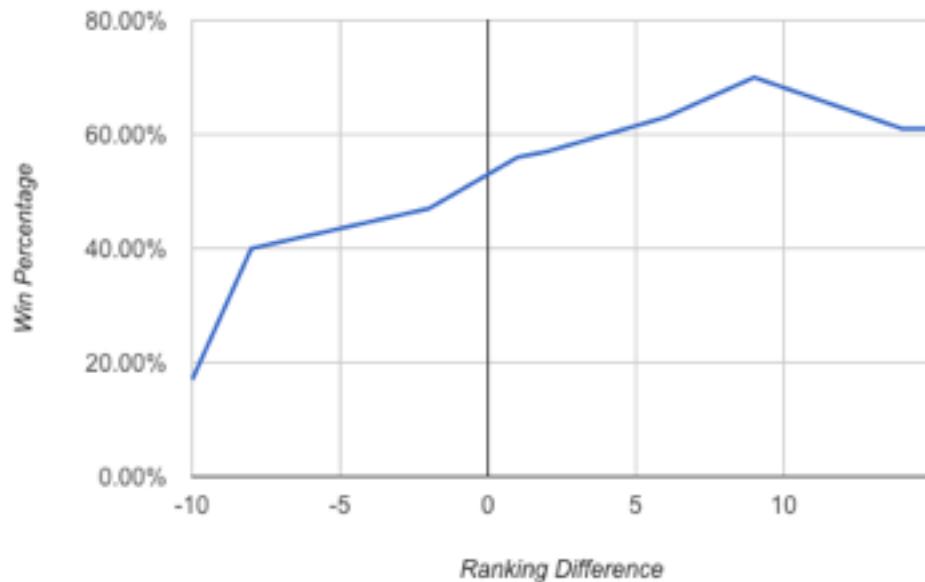
Rank	Team	Total Points	Previous Points +/- Positions	2014 (100 %)		2013 (50 %)		2012 (30 %)		2011 (20 %)	
				Avg.	AVG WGT	Avg.	AVG WGT	Avg.	AVG WGT	Avg.	AVG WGT
1	 Germany	1669 (1668.83)	1765	0	⬇️	1038.02	1038.02	566.64	283.32	751.00	225.30
2	 Argentina	1565 (1564.51)	1631	0	⬇️	949.24	949.24	583.64	291.82	809.86	242.96
3	 Colombia	1420 (1420.20)	1488	0	⬇️	790.30	790.30	636.15	318.08	729.69	218.91
4	 Belgium	1388 (1388.08)	1444	1	⬆️	778.13	778.13	718.22	359.11	549.95	164.99
5	 Netherlands	1375 (1374.53)	1456	-1	⬇️	860.24	860.24	562.68	281.34	396.79	119.04
6	 Brazil	1307 (1307.29)	1291	0	⬇️	799.92	799.92	624.11	312.06	391.47	117.44
7	 France	1191 (1191.23)	1202	2	⬆️	748.38	748.38	405.24	202.62	452.27	135.68
8	 Uruguay	1184 (1184.11)	1243	-1	⬇️	622.61	622.61	616.90	308.45	414.32	124.30
9	 Portugal	1175 (1174.69)	1150	2	⬆️	662.42	662.42	440.74	220.37	645.58	193.67
10	 Spain	1119 (1118.61)	1228	-2	⬇️	395.19	395.19	726.83	363.42	763.94	229.18

Better team wins?

- Higher ranked team wins 55% of time in overtime
  - 60% generally
- Seems like football is quite an arbitrary (random) game
- So we use 55% as baseline
- Does difference in ranking matter?

TOTAL		%
Win	Loss	
4	6	40.00%
14	16	46.67%
1	5	16.67%
9	7	56.25%
10	6	62.50%
11	7	61.11%
11	7	61.11%
15	11	57.69%
7	3	70.00%
82	68	54.67%

### Win Percentages by Ranking Difference



- **Regular Overtime: 56%**
- **Golden Goal: 53%**
- **Penalty Shootout: 54%**
- ***Silver Goal: 0.00%***

Silver Goal - Too little data

Golden Goal - Most random

Penalty shootout & Regular Extratime - similar to 90 minutes

Why FIFA changes the rules

## Recurrence - PSO

Let  $p$  = probability that a pk goes in

Let  $w$  = probability that team 1 wins

PSO's are 5 kicks long and, if still tied, then go 1 kick at a time

For the first 5 kicks:

There isn't a true recurrence. There are  $2^{10}$  or 1024 possible outcomes from those 10 kicks and half will have team 1 win.

For the kicks after 5:

$$w = p^2w + (1-p)^2w + p(1-p) \text{ So } w = 1/2$$

```
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([['score', 'score', 'miss', 'score'], ['miss', 'miss', 'score', 'miss'], 3, 2, 5)
```

- 61% of the time the team that scores first wins
- 93% of the time that it's a shot to win the match, the player scores
- 52% of the time that it's a shot to lose the match, the player scores

## Recurrence - Golden Goal

Let  $p$  = probability that the team with the ball scores

Let  $w$  = probability that team 1 wins

For Golden Goal:

$$w = p + (1-p)^2 w$$

## Recurrence - Golden Goal

$$w_3 = p + (1-p)^2 w_3$$

So ...

$$w_3 = 1/(2-p)$$

Thus the first team with the ball in Golden Goal is guaranteed more than a 50% chance to win, according to the recurrence relation. The first team with the ball is determined by a coin toss.

